Name:

Computer Science 260

Final

Open book, 3 hours

Dec. 19, 2000

Multiple Choice

In the multiple choice questions below, choose *one* alternative only, the one that fits best.

rks

1. Is the relation $f : \{1,2,3,4,5\} \to \{a,b,c\}$ $f = \{(1,a),(2,a),(3,a),(4,a),(5,a)\}$

   (a) a one-to-one function ○
   (b) not a function ○
   (c) an onto function ○
   (d) a one-to-one onto function ○
   (e) None of the above ○

2. The relation $R = \{(1,2),(2,1),(1,1),(2,3),(3,2),(2,2)\}$ on $S = \{1,2,3\}$ is

   (a) reflexive ○ → No
   (b) symmetric ⊗ – is symmetric
   (c) an equivalence relation – if r, [illegible] ○ → No
   (d) transitive ○ → No
   (e) all of the above ○
   (f) both (a) and (b) ○
   (g) both (b) and (c) ○
   (h) both (b) and (d) ○
   (i) none of the above ○

   - reflexive if for every x, (x,x) in relation
   - symmetric if for all x, y (y,x) and (x,y) exist
   - transitive if xRy, yRz and xRz all exist

3. The two premises P and $\neg((P \wedge T) \wedge (P \vee F))$ allow one to conclude

   (a) $F \Rightarrow P$ ○ –
   (b) $(P \wedge Q) \Rightarrow R$ ○
   (c) $R \Rightarrow (P \wedge Q)$ ○
   (d) all of the above ○

   $P, \neg P$
   $F \Rightarrow P$

   $(P \wedge T) \wedge P$
   $P \wedge P$
   $\neg P$

True False Questions

Are the following true or false?

1. The following are contradictions

   (a) $\neg((A \vee B) \wedge \neg(A \vee B))$ – T

   (b) $F \wedge ((A \vee B) \wedge (B \vee A))$ – T

   (c) $\neg(\neg(A \wedge B) \Rightarrow (\neg A \vee \neg B))$ – T

   (d) $(A \Rightarrow B) \wedge \neg A$ → F

2. Given the correct Hoare triple $\{i \geq 0\}C\{j < i\}$. Then one can conclude

   $(j < i) \Rightarrow (j < i+3)$

   (a) $j > 0$ → F

   (b) $\{i \geq 0\}C\{j < i + 3\}$ → T

   (c) $\{i > 2\}C\{j < i\}$ → T

   (d) $\{i > 2\}C\{j < i + 3\}$ → T

   (e) $\{j \geq 0\}C\{i < j\}$ → F

3. paper(book).
   member(X,[X | _]).
   member(X,[_ | Z]) :- member(X,Z).
   append([ ], X,X).
   append([X | Y], Z, [X | A]) :- append(Y,Z,A).
   Relative to the above Prolog database, the following queries succeed?

   tries to compare a list and an element

   fails b/c: member([a], [a|_])

   list a ⇒ [a], not member of [a, b], but is a member of [[a], b]

   T (a) paper(X). → T

   F (b) member([a], [a,b]). → F

   T (c) member(book, [a, book, b]). → T

   F (d) member([a,b,c], a). → F

   (e) append([a,b,c], [c,d,e], [a,b,c,d,e]).

   (f) append([a,b], c, [a,b,c]).

   (g) append([a | [b,c]], [d | [e]], [a,b,c,d,e]).

   (h) append([book], [a,b,c], [book | [a,b,c]]).

Longer Answers

1. Given the set $A = \{1, 2, 3, 4\}$ and the partition of $A\{\{1, 3\}, \{2, 4\}\}$ give the induced equivalence relation on A.

2. Give a formal proof of $A \Rightarrow B$, given the premise $(A \wedge B) \Leftrightarrow A$ For each step of the prrof, give the reason for the step and the numbers of any previous steps referred to.

3. A Prolog data base contains two types of facts: flights and arrivals A fact `flight(ac, 102, saskatoon, toronto)` indicates that airline ac flight number 102 flies from saskatoon to toronto. A fact `arrival(849, 1605, 69)` indicates that a flight with number 849 arrives at its destination at time 1605 at gate 69. There are many facts of each type in the database.

Write Prolog queries that will extract the following information from the database.

(a) The name of an airline that flies to saskatoon.

flight (airline, _, _, saskatoon)

(b) The name of a city at which a flight arrives at time 1605 at gate 69.

flight (_, Num, _, City), arrival (Num, 1605, 69)

(c) The flight number of a flight that leaves from saskatoon, and arrives at its destination after 2200.

flight (_, Num, saskatoon, _), arrival (Num, X, _),

X >= 2200.

4. Let P and Q be subsets of the universal set E. Prove that if $x \in A$ where $A = (\sim(\sim P \cap Q) \cup P) \cap \sim Q$, then $x \in C$ where $C = (P \cup Q) \cup (\sim P \cap \sim Q)$.

$A = (\sim(\sim P \cap Q) \cup P) \cap \sim Q$

$= (P \cap Q) \cup P \cup Q$

$= \sim P \cup \sim Q \cup P \cup Q$

$A =$

$= ((P \cup \sim Q) \cup P) \cap Q$

$= (P \cup \sim Q) \cap Q$

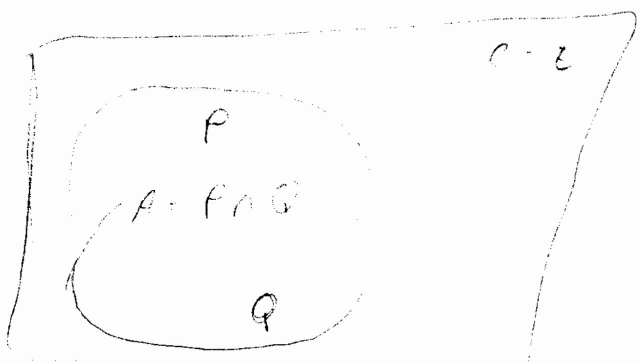
$(P \cap Q) \cup \underbrace{(\sim Q \cap Q)}_{\emptyset}$

$= P \cap Q$

$C = (P \cup Q) \cup (\sim P \cap \sim Q)$

$C = (P \cup Q) \cup \sim(P \cup Q)$

$C = E$



5. Write a Prolog procedure recount that has two arguments. The first argument is a list of structures, *result(poll#, gorevote, bushvote)* where *poll#* is a poll identification number, *gorevote* is the number of votes obtained by candidate gore and *bushvote* is the number of votes obtained by candidate bush. The procedure recount should succeed if the second argument is a list of poll identification numbers for polls in which the difference between the *gorevote* and the *bushvote* is less than 100. The numbers in the second list should appear in the same order as they do in the first list.

   Example:

   recount([result(14,502,714), result(7,812,813), result(18,1100,1050), result(5,1000,100)], [7,18]). should succeed.

```
recount([result(PollNum, G, B)|T], List) :- A is G-B,
  ((A ≥ 0, A < 100); (A ≤ 0, A > -100)), member(PollNum, List),
  recount(T, List).
recount([H|T], List) :- recount(T, List).
recount([], _).
```

6. Complete the following proof by inserting the missing lines and justifications (including line numbers). Prove $\exists z \forall y \exists w((P(z,y) \vee \neg Q(w)) \wedge R(x))$ given the premise $\exists x(\forall y P(x,y) \vee \neg \forall y Q(y)) \wedge R(x)$.

1. $\exists x(\forall y P(x,y) \vee \neg \forall y Q(y)) \wedge R(x)$	premise
2.	1, rule 2 and rule 2d
3.	2, rule 4d
4. $\exists z((\forall y P(z,y) \vee \exists w \neg Q(w)) \wedge R(x))$	
5. $\exists z(\forall y(P(z,y) \vee \exists w \neg Q(w)) \wedge R(x))$	
6. $\exists z(\forall y(P(z,y) \vee \exists w \neg Q(w)) \wedge \forall y R(x))$	
7.	6, rule 5
8.	7, rule 1d
9. $\exists z \forall y(\exists w(P(z,y) \vee \neg Q(w)) \wedge R(x))$	
10. $\exists z \forall y \exists w((P(z,y) \vee \neg Q(w)) \wedge R(x))$	

7. Do a complete correctness proof for the following program. Given the precondition {n integer, n ≥ 0 } and the postcondition { n integer, n $\geq 0, z = n^2(n+1)^2$ } Give the preconditions and postconditions of each statement, using the space provided. Also, prove that the loop invariant together with the negation of the entry condition $i <> n$ implies the postcondition.

Precondition	Statement	Postcondition
	$i := 0;$	
	$z := 0;$	
	while $(i <> n)$ do	
	begin	
	$i := i + 1$	
	$j := i * i;$	
	$k := j * i;$	
	$z := z + k;$	
	end	
	z := z * 4;	

8. A data base contains two relations, called PARTS and STOCK. The header for PARTS is

   Part_name   Unit_price   Part_Id#   Supplier

   Similarly, the table STOCK has the header

   Part_Id#   Quantity

   Use relational algebra to obtain

   (a) The names of all parts that cost more than $100 each.

   select Part_name from PARTS where Unit_price ≥ 100

   (b) A list of the suppliers who supply parts that are presently in short supply. That is, the quantity in stock is less than 5.

   (c) A list of the Part_Id#s for parts, that cost more than $50 each, and for which the quantity in stock is greater than 10.

Is your name on the cover?

Total

The End

((COMMITTEE [com_name = 'Research']) [com#] * MEMBER_OF) [emp#]) * FACULTY